Generalized Deferred Acceptance Auctions with Multiple Relinquishment Options for Spectrum Reallocation

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It is really a great pleasure to have an opportunity to present in this seminar. Today I would like to talk about my projects concerning the design of market-based allocation mechanisms for spectrum reallocations.

The organization of the talk is:
- The Statement of the Problem
- Previous Results
- The Approach of the Paper and the Results
- Contributions of the paper.
I. Introduction.

- Progress in Mobile Broadband Technology.
- Public Policy Agenda for Spectrum Reallocation.
- FCC Incentive Auctions.
- The Topic of the Paper: Reverse Auctions.
- The Key Feature: Assignment Problem with Interference Constraints.
- The State of the Art: Milgrom and Segal Deferred Acceptance Algorithm.
- The Research Question: Multiple Relinquishment Options.
- The Main Result: Generalized Deferred Acceptance Auctions with the Supplementary Phase.
PROGRESS IN MOBILE BROADBAND TECHNOLOGY

- There have been rapid progresses in the mobile broadband technology that allows for any time, anywhere access of high-bandwidth applications, contents, and communications.
- The first wireless Internet access in 1991, offered the speed 237Kbits/second (GSM Edge).
- The current fourth (4G) generations technology delivers 100–300Mbits/second (LTE Advanced).
**Wide Adoption of Mobile Communication**

- These progresses in mobile broadband technology led to wide adoption of mobile communication devices that rely on extensive uses of spectrum.
- In 2012, there are 172 million smartphone subscribers.
- More than 29% of adults own tablet computers up from 3% in 2009 in the United States.
- The average amount of traffic per smartphone in 2012 is 342 MB per month, up from 189 MB per month in 2011.
- Global mobile traffic is projected to grow 13 folds between 2012 and 2017, reaching 11.2 exabytes per month.
SIGNIFICANCE OF MOBILE BROADBAND TECHNOLOGY

• Mobile Communications are *general purpose technology* that can affect our daily lives and industry productivity.
• Examples include the steam engine, railroad, electricity, electronics, the automobile, and the computer.
• As such, it is politically important to develop infrastructure for the mobile broadband technology.
PUBLIC POLICY AGENDA FOR REALLOCATION OF SPECTRUM

Given these surge in demand for mobile communications, reallocation of spectrum from current licensees TV broadcasters to mobile uses has become a public policy agenda.

- Currently, there are 8402 broadcasters operating in 3 VHF (very high frequency) bands and 2 UHF (ultra high frequency) bands.
- Each broadcaster has been assigned to a 6MHZ block of spectrum covering a specific geographic area.
- Given the technological characteristics of spectrum of the UHF bands, the FCC focuses on reallocations of spectrum in the UHF band.
CURRENT ALLOCATION OF SPECTRUM TO TV STATIONS
REALLOCATION OF THE UHF BAND

- The Current Allocation.

- Planned Allocation.
The Congress authorized FCC to conduct incentive auctions in the Spectrum Act in February 2012.

Incentive auctions are voluntary, market-based means of repurposing spectrum by encouraging licensees to relinquish spectrum usage rights in exchange for a share of the proceeds from an auction of new licenses to use the repurposed spectrum.
ITS FUNDAMENTAL STRUCTURE

- The FCC Report and Order FCC 14-50 determines the structure.
- The first step is *Reverse Auctions*: TV stations submit bids to voluntarily relinquish spectrum rights to FCC in exchange for payments.
- Then *Repacking* is a reorganization of the broadcast television bands where television stations that remain on-the-air will occupy a smaller portion of the UHF band.
- The *Forward Auctions* identify the prices that potential users of repurposed spectrum would pay for the new licenses.
The Key Idea

- Broadcasters relinquish spectrum usage rights for compensation.
- Mobile Broadband providers bid for spectrum licenses.

Broadcasters
Reverse Auction

Mobile Broadband Providers
Forward Auction
THE TOPIC OF THE PAPER: THE DESIGN OF REVERSE AUCTIONS

• This paper, we focus on the design of reverse auctions.
• The design of reverse auctions that lead to active participation by TV stations is crucial to achieve the objective of reclaiming large amounts of spectrum and raising revenues.
• FCC Chairman Wheeler notes in June 2014 that “robust participation by broadcasters will be crucial to the success of the auction.”
In the assignment problem, there are two kinds of agents (buyers and sellers) and each seller owns one unit of the indivisible commodity, each buyer has no use for no more than one unit, and utility is transferable and side payments are allowed.

Its early example is the real estate market.

Kuhn, Koopmans and Beckman, Shapley and Shubik are pioneering literatures on this topic.

The case where sellers have many units (“many-to-one matching markets”) has also been extensively studied in the context of sponsored search auctions (for example, Varian).
**The Key Feature: Interference Constraints**

- The key features in incentive auctions, which are not present in the standard assignment problem, are interference constraints among stations.
- Interferences occur when
  - The two frequencies are close on the Electromagnetic band.
  - Connections are geographically close to each other, so that interfering signals are powerful enough to disturb the quality of a signal.
- This interference constraints lead to the Boolean Satisfiability Problem that is NP-Complete.
AN EXAMPLE OF TV STATIONS IN NORTHEAST CORRIDOR

- This is an example of 2 TV Stations Operating in the New York Market (WABC-TV in Channel 7 and WMBC-TV in Channel 18)
TV STATIONS CONTOURS
INTERFERENCE AMONG TV STATIONS
**The Key Idea: Adjusting Values with Scores**

- These interference constraints make the standard assignment algorithms infeasible to the reverse auctions problem.
- The idea is to use the scoring function that encodes these interference constraints.
- By adjusting the values with the scores, the reverse auction problems can be transformed into the standard assignment problem.
Milgrom and Segal (2014) develop Deferred Acceptance Auctions when a seller has only one selling options.

- An offer can be evaluated with its scoring function that encodes interference constraints.
- Allocation rule is to accept an offer with the low. The buyer rejects the offer with the highest score.
- Then the station with the rejected offer will be repacked.
- Then offers are re-evaluated.
- Auctions end when only offers with zero scores remain.
- The payment rule is determined by the threshold price that is the maximum amount that the seller can offer to sell.
Properties of Deferred Acceptance Auctions

- When bidders are single-minded (that is, the seller is interested in only one item), then deferred acceptance auctions are
  - strategy-proof
  - can be implemented in descending clock auctions,
  - nearly-optimal,
  - group strategy-proofness,
  - equivalent to pay-as-bid auctions
The Milgrom and Segal (2014) analysis assumes single-minded bidders. But the Spectrum Act 6403(a)(2) stipulates that bidders shall have multiple relinquishment options such as going off the removing to the VHF band, and channel sharing.
**Broadcaster Options**

- **Go Off Air:** Bid to relinquish license, receive payment, and go off the air.
- **UHF to VHF:** Bid to relinquish a UHF channel, receive payment, and move to either a high VHF (7 to 13) or low VHF (2 to 6) channel.
- **Channel Sharing:** Bid to relinquish current channel, receive payment, and share a channel with another broadcaster.
STRATEGIC MANIPULATIONS IN THE CURRENT DEFERRED ACCEPTANCE AUCTIONS

- When the seller can make only one offer, the seller has to choose which auctions to bid.
- And the seller’s choice of auctions depends on the payment the seller will get.
- Then the payment depends on the choice of auctions by other bidders.
- Therefore, the seller’s choice of auctions can depend on other sellers’ choices and sincere bidding may not realize.
- Allowing multiple offers can resolve this question.
Then the research question of the paper is to define a generalization of deferred acceptance
- to allow multiple relinquishment options
- establish its strategy-proofness
- construct equivalent descending clock auctions.
THE EFFECT OF INTERFERENCE CONSTRAINTS

- When stations can interfere each other, switching across relinquishment options can create interferences with stations that are already assigned to this band.
- The standard assignment mechanisms (Demange, Gale, and Sotomayor based on Kuhn’s Hungarian algorithms and Crawford and Koner based on deferred acceptance) are infeasible since these standard mechanisms assume that assignments are possible as long as they satisfy the quantity bounds.
AN EXAMPLE

• Suppose the buyer runs deferred acceptance auctions simultaneously both for the UHF band and for the VHF band.
• Now consider a station has an offer of going off the air rejected.
• Since the offer of going off the air is rejected, the station would very well consider switching its offer to the moving to the VHF auction.
• With interference constraints, switching to the VHF band may be infeasible.
• This can happen even if the station offers to move to the VHF band with a very low offer.
Main Results: Generalized Deferred Acceptance Auctions

- We consider a generalization of deferred acceptance auctions.
- A seller is allowed to make multiple offers for multiple options.
- An offer in each option is scored according to the interference with TV stations already assigned in each option.
- A seller cannot make new offers in the middle of the auction or reenter the auction the seller previously dropped out.
- The price of an offer for an option is the maximum offer that the seller can make to sell through the option.
- The supplementary phase assigns the seller to the option that provides the seller the highest profit.
ADVANTAGE OF THE GENERALIZED AUCTIONS

- Since a seller can make multiple offers for multiple options from the beginning, the seller does not need to switch across options in the middle of the auction.
- Since a seller does not need to switch and does not make new bids in the middle of the auction, the buyer does not need to adjust the interference constraints.
- Finally, the final allocation satisfies interference constraints.
AN EXAMPLE (CONTINUED)

- In the setting of the previous example, the station can make offers in both going off the air auctions and moving to the VHF band auction at the same time in the first auction phase.
- In supplementary phase, the seller is assigned to the auction that will provide the seller the maximum profit.
- Thus a seller does not switch across auctions in the process.
EQUILIBRIUM OF THE GENEARLIZED DEFERRED ACCEPTANCE AUCTIONS

• When sellers have multiple sell options but a unit supply, deferred acceptance auctions are strategy-proof.
• Auctions can be implemented as a two-stage mechanism. This format simplify participation for broadcasters and facilitate price discovery through information sharing among participants.
• Auctions inherit the properties of deferred acceptance auctions, can realize high efficiency levels, robust to group deviation, and equivalent outcomes with pay-as-bid auctions.
**Contributions of the Paper.**

- Theoretically, this paper shows that the deferred acceptance auctions can be extended to the practically important economic environment that allows multiple relinquishment options for a seller.
- Practically, in terms of the FCC incentive auctions setting, the paper shows that the deferred acceptance auctions of Milgrom and Segal (2014) can be generalized to accommodate the multiple seller relinquishment options.
II. Reverse Auctions Economic Environments

• The model of this paper is based on FCC Report and Order FCC 14-50.
• The model and generalizes the Shapley-Shubik assignment model with interference constraints.
• It also generalizes the [?] model with multiple relinquishment options for potential sellers.
THE ELEMENTS OF ECONOMIC ENVIRONMENTS

- The Buyer
- Sellers
- Unit Supply
- Relinquishment Options
- Private Values
- Preferences
- Interference Constraints
There is a single buyer in this economic environment. 
FCC 14-50 states that the FCC identifies a set of bidders that would voluntarily relinquish spectrum usage rights and the compensation that each would receive.
**POtential selErls**

- Let $i = 1, \ldots, I$ denote potential sellers. Let $N$ be the set of potential sellers.

- The FCC Report and Order states that full power and class A stations are eligible to participate in the reverse auctions. We consider these sellers in the UHF band.

- A license is entitled to be protected in the repacking process if they do not file for participation.
UNIT SUPPLY

- We assume that each seller has a unit of indivisible goods that the seller can choose to relinquish.
- The FCC Report and Order states that a winning reverse auction bidder relinquishes all usage rights for that channel and would retain no further rights with regard to the channel.
- This assumption is in line with the standard assignment problems.
- In the previous FCC auctions, each bidder (mobile operator)’s value for a given license depends upon the other licenses it obtains.
**Relinquishment Options**

- Let $j = 1, \ldots, J$ be a bid option through which a seller can relinquish the goods.
- The FCC Report and Order states that the reverse auction shall include 3 relinquishment options for participants.
  - The first option is going off the air (“license relinquishment bid”).
  - The second option is “UHF-to-VHF bids.”
  - The third option is “Channel Sharing Bids.”
**Private Values**

- Let $c_{i,j}$ be the private cost and expenses that seller $i$ incur from selling a good through option $j$.
- In the terminology of the assignment problems, $(c_{i,j})_{i=1,...,n,j=1,...,J}$ is the rating matrix of station $i$ for channel $j$.
- In FCC reverse auctions, $c_{i,1}$ is the cost of a station from going off the air by forgoing all future revenues from its operation as a TV broadcaster.
- $c_{i,2}$ refers to the cost of moving to a lower VHF band with some additional adjustment costs due to a limited range of VHF digital signals compared with UHF digital signals.
- $c_{i,3}$ involves the cost from moving to a different band by sharing.
We assume that sellers’ preferences are quasi-linear and risk-neutral.

In other words, utility is identified with money.

Let $p_{ij}$ be the compensation that seller $i$ receives from selling through the $j$th option.

Then seller $i$ has payoff of $p_{ij} - c_{ij}$ when the seller sells the good through the $j$th option and 0 otherwise.
**Interference Constraints**

- For the UHF band, rejecting an offer and reassigning the station in the UHF band can create interferences with existing stations in the new UHF band.
- For the VHF band, accepting an offer so that a station can move to the VHF band can create interferences with existing stations in the VHF band.
- For both UHF and VHF band, assignment can create interferences with TV whitespace uses (wireless medical telemetry, radio astronomy, wireless microphone, community broadband services, IEEE 802.11AF).
III. Generalizing the Deferred Acceptance Auctions

- We now discuss our approach of extending deferred acceptance auctions to allow multiple relinquishment options.
- Milgrom-Segal example
- Allowing multiple relinquishment options
- Need for the Supplementary Phase
**Example 1 (Milgrom and Segal)**

- Suppose that the buyer has 1 unit to buy.
- There are 3 potential sellers with costs 10, 5, and 4.
- Suppose that these sellers offer truthfully.
- Then the threshold prices for these sellers are 4, 4, and 5.
- Then the payoff for these sellers are -6, -1, and 1.
- Thus the allocation rule is interpreted as allocating to the seller whose provisional profit, that is, (the threshold price) - (the cost), is positive.
STRATEGY-PROOFNESS

• With the threshold pricing, for all offer profile such that threshold price is greater than the cost, it is a best response to offer truthfully.
• Also for all offer profiles such that the threshold price is less than cost, it is a best reply to offer truthfully and not to sell the good.
• Consequently, truthful offers are a dominant strategy.
Suppose a seller has multiple options through which the seller can relinquish a unit of goods.

Then, one generalization of the previous approach is that the seller makes offers for multiple options, the buyer calculates the threshold price for each seller for each option, and the buyer makes a take-it-or-leave-it offer for each seller and for each option.
EXAMPLE 2 (MULTIPLE RELINQUISHMENT OPTION)

- Each station has 2 options (going off the air or moving to the VHF band) to relinquish a unit of goods.
- The buyer can accommodate at most 2 stations in the new UHF band plan and at most 1 station in the VHF band.
- Suppose there are 3 potential sellers.
- Their costs are as follows:

<table>
<thead>
<tr>
<th>Costs</th>
<th>Going Off the Air</th>
<th>Moving to the VHF Band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station 1</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Station 2</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Station 3</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>
Suppose that sellers make truthful offers. Then the threshold prices are:

<table>
<thead>
<tr>
<th>Threshold Prices</th>
<th>Going Off the Air</th>
<th>Moving to the VHF Band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seller 1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Seller 2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Station 3</td>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>
Then the profits (\(=\text{threshold prices} - \text{costs}\)) are given by the table below and the assignment is: seller 1 stays in the UHF band, seller 2 moves to the VHF band, and seller 3 goes off the air.

<table>
<thead>
<tr>
<th>Threshold Prices-Costs</th>
<th>Going Off the Air</th>
<th>Moving to the VHF Band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station 1</td>
<td>-6</td>
<td>-6</td>
</tr>
<tr>
<td>Station 2</td>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>Station 3</td>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>
NEED FOR ASSIGNMENT PHASE

- In the previous example, the parameter values are such that each seller has at most 1 relinquishment option that provides a positive profit.
- But it is possible that a seller has low costs for both options and that the seller can have positive payoffs from selling through multiple options.
- But the seller cannot sell more than one unit of good. In this case, the buyer needs to conduct a supplementary phase.
- Allowing a seller to sell through the option with the highest profit will provide the seller the incentive to report the cost truthfully.
Example 3 (Assignment Phase)

Now suppose that the cost structure is such that seller 3 has low costs for both options.

<table>
<thead>
<tr>
<th>Costs</th>
<th>Going Off the Air</th>
<th>Moving to the VHF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station1</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Station 2</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Station 3</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>
In this case seller 3 has a largest profit from going off the air. Thus the final assignments are that seller 1 and 2 stay in the UHF band, and seller 3 goes off the air.

<table>
<thead>
<tr>
<th>Threshold prices - costs</th>
<th>Going Off the Air</th>
<th>Moving to the VHF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station 1</td>
<td>-6</td>
<td>-6</td>
</tr>
<tr>
<td>Station 2</td>
<td>-3</td>
<td>-1</td>
</tr>
<tr>
<td>Station 3</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
INTERFERENCE CONSTRAINTS

• Suppose a station has both bids rejected. The station is repacked to the UHF band. Its feasibility is checked at the time the bid for going off the air was rejected.

• If a station has going off the air bid accepted and that the bid for the VHF band rejected, then the station will go off the air. This assignment does not violate the interference constraints in the VHF band.

• Suppose that a station has a going off the air bid rejected and the bid for the VHF band accepted. In this case, the station will move to the VHF band. The interferences with stations at the VHF band are already checked at the auction phase.
Case where a Station is a Provisional Winner in Both Auctions

- Finally, suppose that a station has both offers accepted.
- If the station turns out to be going off the air, removing stations from the VHF band will not create new interferences at the VHF band.
- If the station turns out to be or moving to the VHF band, the interference with stations at the VHF band is already checked at the auction phase.
GENERALIZED DEFERRED ACCEPTANCE AUCTIONS

Station Offers

Descending clock auctions

Repacked at UHF

Interferes with UHF stations, Relocate to VHF

Efficient Relocation of stations

No Interferences with VHF stations

Interferences with VHF stations

Moving to VHF

Going off the air

Going off the air
IV. Formal Statement of the Mechanism and Strategy-Proofness

- The Bid Collection Procedure.
- The Auction Phase.
- Scoring Function
- Auctions for Going Off the Air.
- Auctions for “Moving to the VHF Band.
- The Supplementary Phase.
- Strategy-Proofness
**The Bid Collection Procedure**

- Each seller, for each relinquishment option, offers the payment that the seller is willing to accept in exchange for relinquishing the goods through the option.
- Formally, let an offer made by seller \( i = 1, \ldots, n \) for the \( j \)th option be \( b_{i,j} \).
- Let the set of possible bids by seller \( i \) for option \( j \) be \( B_{i,j} \).
- We assume these sets are finite and actually the same across all bidders and options: there exists \( B \) such that \( B = B_{i,j} \) for all \( i \) and \( j \).
- The profile of bids for the \( j \)th option is \( b_j = (b_{1,j}, \ldots, b_{n,j}) \).
The Auction Phase

- The auction phase determines which bids are acceptable and which bids are rejected.
- In deferred acceptance auctions, a bid is evaluated with the scoring function that is a function of other bids for that option.
- Let $A_j$ be the set of active bidders in the auction for the $j$th option.
- Let $N \setminus A_j$ be the set of “frozen” bidders in the auction for the $j$th option.
- In the going-off-the-air auction, $N \setminus A_j$ is the set of bidders whose bids are rejected and repacked at the UHF band.
- In the auctions for moving to the VHF band, $N \setminus A_j$ is the set of bidders whose bids are accepted and assigned to the VHF band.
The Scoring Function

• Then the scoring function of seller $i$ for the $j$th option is $s_i^A_j : B_{i,j} \times B_{N\setminus A_j} \rightarrow \mathbb{R}_+$ that is nondecreasing in the first argument.
• Here the scoring function for seller $i$ for the $j$th option depends only on the bids for the $j$th option.
• For example, in the auctions for going off the air, the scoring function checks interferences with already repacked stations at the UHF band.
• If there are interferences with existing UHF stations, then the score would be zero, the offer would be accepted, and the station would go off the air.
Auctions for Going Off the Air

- The auctions for “the going off the air” option are the deferred acceptance auctions of Milgrom and Segal (2014). Let this option be \( j = 1 \).
- The bids for this option is \( b_1 = (b_{1,1}, \ldots, b_{n,1}) \).
- The scoring function for seller \( i \) is \( s_i^{A_1}(b_{i,1}, b_{N \setminus A_1,1}) \) that depends on seller \( i \)’s offer for the 1st option and bids for the \( j \)th option by sellers already repacked to the UHF band.
Auction Stages

- The auction at stage $t$ works as follows. Let $A_{1,t} \subseteq N$ be the set of active bidders for option 1 at stage $t$.
- Then, let $A_{1,t+1} = A_{1,t} \setminus \arg \max_{i \in A_{1,t}} s_{i}^{A_{1,t}}(b_{1}, b_{N \setminus A_{1,t}})$.
- That is, the buyer removes the seller with the highest score.
- That seller (the station) is repacked at the UHF band.
Termination of Auctions

• The auction stops at the first stage $t'$ such that $s^A_{i, t'}(b_{i, 1}, b_{N\setminus A, t'}) = 0$ for all $i \in A_{1, t'}$. That is, all remaining stations have zero scores and cannot be repacked.

• The threshold price for bidder $i$ is $p_{i, 1}(b_1) = \sup\{b_{i, 1} : i \in \alpha_1(b'_{i, 1}, b_{-i, 1})\}$. That is, the maximum price that bidder $i$ can offer to sell through the 1st option given the offers by other sellers.
Auctions for Moving to the VHF Band.

• In the auctions for offers for moving to the VHF band, the buyer needs to check interference constraints with the station to the VHF band.

• Then the auctions start with accepting offers with low scores and stop when it is no longer possible to accept stations because of interference constraints.
The Supplementary Phase.

- The above auction phases make provisional assignments of stations for each band based on bids for each band.
- The auction phase does not take into account of the quantity constraint that each seller can supply only one unit of good.
- Then the supplementary phase determines the final allocation.
**Determining the Final Allocations**

- Let \( \alpha_j(b_j), j = 1, \ldots, J \) be the set of provisional winners for each option.
- If \( \alpha_j(b_j) \cap \alpha_{j'}(b_{j'}) = \emptyset \) for all \( j \) and \( j' \), then, there are no overlaps of winners across auctions.
- Otherwise, there exists some sellers who are winners in multiple auctions.
- Let \( i' \) be a such seller and let \( j_1, \ldots, j_k \) be options such that \( i' \in \alpha_{j_1}(b_{j_1}), \ldots, i' \in \alpha_{j_k}(b_{j_k}) \).
- Then, seller \( i' \) is assigned to the option \( j^* \) such that \( j^* \in \arg \max_{j' \in \{j_1, \ldots, b_{i'}, j'\}} \). That is, \( j^* \) is the option that provides the highest profit for seller \( i' \).
In the generalized deferred acceptance auctions, sellers bid for multiple options to sell. Since the buyer would buy through the options that would provide the highest payoff for a seller and a seller’s offer cannot affect the threshold price, the seller’s best response is to make offers truthfully.

**Proposition 1.** In the reverse auctions economic environment, generalized deferred acceptance auctions with the supplementary phase are strategy-proof.
Proof Sketch

• Let us consider seller $i$’s offer for the $j$th option.
• Consider the case where seller $i$ has the highest provisional profits with the $j$th option among all options when the seller $i$ bid truthfully.
• In this case, seller $i$ sells from truthtelling and obtains the positive profit.
• Even if seller $i$ lowers the offer, it will not affect the outcome.
• If the seller increases the offer, since seller $i$ may not be able to sell through the $j$th option anymore, the profit may decrease.
THE OTHER CASE

• When seller $i$ does not have the highest provisional profits with the $j$th option with truthtelling, seller $i$ does not sell from the $j$th option with truthtelling.

• Lowering an offer may lead to an outcome of selling through the $j$th option that will reduce the payoff.

• If the seller increases the offer, the outcome will not change.
V. IMPLEMENTATION WITH DESCENDING CLOCK AUCTIONS

• The previous section presented simultaneous in a single round sealed bid procedure.
• The FCC Rules and Order states that the second option for reverse auctions is a multiple round or dynamic process in which bidders indicate the willingness to accept lower payment in exchange for relinquishing rights.
• This section provides such a dynamic implementation of the generalized deferred acceptance auctions.
• Simultaneous Descending Clock Auctions.
• Supplementary Phase
• Equivalene with the Generalized Deferred Acceptance Auctions
The Price Clock

- The first stage can be considered as running descending clock auctions of Milgrom and Segal (2014) for each option.
- Let $A_{j,t} \subset N$ be the set of active bidders for auction $j$ at stage $t$. The profile of set of active bidders for all auctions is $A_t = (A_{1,t}, \ldots, A_{J,t})$.
- The history of auction $j$ at stage $t$ is the sets of active bidders for auction $j$ up to stage $t$.
- The set of all histories for auction $j$ is denoted by $H_j$.
- The price clock for the $j$th auction is a mapping $p_j : H_j \rightarrow \mathbb{R}^N$ with the property that, for each seller $i$,
\begin{equation}
\left( A_j^t \right) \leq p_{i,j}(A_j^t).
\end{equation}

personalized price clock for seller $i$ in auction $j$
**Simultaneous Descending Clock Auctions**

- At the first stage, we have $A^1_j = N$ for all $j$. That is, in the initial stage, the set of active bidders is the whole set of bidders.
- At each period $t$, for each auction $j$, given history $A^t_j$, each seller $i$ is offered $p_{i,j}(A^t_j)$.
- When $p_{i,j}(A^t_j) < p_{i,j}(A^{t-1}_j)$, seller $i$ can decide whether to stay in the auction or exit (and repacked).
- For each auction $j$, let $E_{j,t} \subseteq A_{j,t}$ be the set of bidders who choose to exit. Then the set of active bidders is $A^{t+1}_j = A_{j,t} \setminus E_{j,t}$.
- Auction $j$ ends at stage $t$ when $p_{i,j}(A^t_j) = p_{i,j}(A^{t-1}_j)$ for all $i \in A^t_j$, that is, when the price clock stops.
- Then the active bidders $i \in A^t_j$ will be provisional sellers with price $p_{i,j}(A^t_j)$.
The Supplementary Phase

- Once the descending clock auctions have ended and when there are sellers who are selected as a provisional seller in multiple auctions, the supplementary phase will assign such sellers to sell through the auction with that provides the highest profit to the seller.
EQUIVALENCE WITH THE GENERALIZED DEFERRED ACCEPTANCE AUCTIONS

• Define the cutoff strategy to be a strategy $b_{i,j}$ such that bidder $i$ exits from the auction if $p_{i,j}(A^t) < b_{i,j}$.

• Proposition 2. For any generalized deferred acceptance auction with a finite set of bids, there exists an equivalent simultaneous clock auctions with cutoff strategies.
Proof Idea

• For each auction $j$, as in Milgrom and Segal (2014), construct a price clock that reduces the prices for sellers with the highest score by the minimum price increments while keeping the price clock fixed for other sellers.

• Then, for every history, the next set of sellers to quit is the set of bidders with the maximum score.

• Thus the outcome of the auction phase in deferred-acceptance heuristic auctions and the outcome simultaneous descending clock auctions are equivalent.

• Then the supplementary phase will lead to the equivalent outcome of the whole auction process.
VI. Further Properties of Generalized Deferred Acceptance Auctions

- We now show that Generalized Deferred Acceptance auctions inherit desirable properties of the Milgrom-Segal Deferred Acceptance Auctions.
- Weak group strategy proofness
- Relationship with Simultaneous Pay-As-Bid Auctions.
Weak Group Strategy-Proofness

• Milgrom and Segal employ a notion of weak group strategy-proofness.
• It implies that, for any group deviation, there is at least one seller who does not strictly gain from the group deviation.
• Formally, it says, for every profile of values $c$, for every set of players $S \subset N$, and for every strategy profile $\sigma_S$ of these players, at least one bidder in $S$ has a weakly higher payoff from the profile of truthful bids $c$ than from the strategy profile of players in a set $S$ players according to the strategy profile $\sigma_S$ and other players behaving truthfully.
Weak Group Strategy-Proofness of Generalized Deferred Acceptance Auctions

- Milgrom and Segal (2014) show that the deferred acceptance auctions are weakly group strategy proof.
- Indeed, in generalized deferred acceptance auctions, the seller’s profit is the maximum of the profits from each auction.
- Thus, when a seller has a deviation from one auction, a seller prefers to use this deviation in that auction when the seller chooses the bidding strategy in overall auctions.

Proposition 3. Generalized deferred acceptance auctions with the supplementary phase is weakly group strategy-proof.
EQUVALENCE TO SIMULTANEOUS PAY-AS-BID AUCTIONS

• The previous sections consider generalized deferred acceptance auctions with the supplementary phase and study their properties. An alternative method is pay-as-bid auctions.

• Milgrom and Segal (2014) compare these two mechanisms and derive the equivalence between these two auction mechanisms: the pay-as-bid auctions are dominance solvable and a Nash equilibrium of the auction game is the equivalent to the one in the deferred acceptance auctions.

• The intuition is that an iterated deletion of weakly dominated strategies is closely related to the deletion of least favorable offers in the deferred acceptance algorithm.
**Equivalence of Generalized Deferred Acceptance Auctions**

- The generalized acceptance auctions are a combination of deferred acceptance auctions and the supplementary phase where the seller chooses the most favorable outcome.

**Proposition 4.** Consider a simultaneous paid-as-bid auction with the supplementary phase. In this case, one strategy profile that survives an iterated deletion of dominated strategies and is a Nash equilibrium in undominated strategies is the one equivalent to the equilibrium in the Generalized Deferred Acceptance Auctions.
VII. Conclusions

- Main Results
- Contributions
- Future Research
This paper studies the design of reverse auctions in the US incentive auction process. The state of the art is the generalized deferred acceptance auctions of Milgrom and Segal (2014).

A significance of this research question is that allowing multiple options can increase efficiency, revenue, and encourage participation in auctions.

A main difficulty is that just running simultaneous auctions is not feasible since seller switching across auctions will create interferences with stations already assigned to the band.
Main Results of the Paper

- We propose generalized deferred acceptance auctions with the supplementary phase.
- First, the auctions are strategy-proof because of the threshold pricing and also because the seller can maximize the profits from auctions.
- Second, the auctions can be implementable. in descending clock auctions with the supplementary phase by developing a correspondence between the scoring function and the descending price clock.
- Third, auctions are nearly-optimal, weakly group
- It is because of the environment specific properties that a seller has a unit supply.
CONTRIBUTIONS OF THE PAPER

- The contribution of the paper is the strengthening of the results of Milgrom and Segal (2014).
- This generalization can be useful in related reallocation and sharing of the government spectrum and other complex allocation problems.
- A future research agenda is the integration of scoring methodology and the assignment problems.
- A long-term research and policy for spectrum allocation will include dynamic spectrum allocation and open/hierarchical access to spectrum based on cognitive radio technology.